DHMS Math Club Meeting #2 October 1, 2013

We discussed probability, factorials, prime numbers, and last week's question set.

Probability rules:

If two events (A and B) are **mutually exclusive**, they cannot occur at the same time (cannot both be true at the same time). In this case,

Probability of A OR B = probability of A + probability of B

Example: At DHMS, 30% of the students are in 6th grade and 36% of students are in 7th. If you choose a random DHMS student, what is the probability that the student is a 6th grader or 7th grader?

A student cannot be a 6th grader and a 7th grader at the same time. These are mutually exclusive cases. The required probability is 0.30 + 0.36 = 0.66

Two events are **independent** if they do not affect each other. For independent events A and B, the probability of A AND B occurring = probability of A * probability of B.

Example: The probability that I will rain tomorrow is 0.2. The probability that Brian's dog will eat his homework tomorrow is 0.1. What is the probability that tomorrow it will rain and Brian's dog will eat his homework?

These are independent events (*unless rainy weather make's Brian's dog develop a hankering for his homework*).

The required probability = 0.2*0.1 = 0.02.

Factorials:

We use a math concept called factorial to calculate **permutations** and **combinations.** Some examples: How many ticket numbers are possible in a certain lottery?

How many combinations of desserts can you make with eight ice cream flavors, seven toppings, and four sauces if a dessert must have one scoop of ice cream, one topping and one sauce?

How many outfit combinations are possible with the clothes in your closet?

How many different ways can you arrange the letters of your name?

The factorial of n (for n > 0) is the product of all integers from 1 to n. It is represented by the number followed by an exclamation point. By convention 0! is 1, because there is only one way to arrange zero objects. Negative integers do not have factorials.

1 factorial = 1! = 1

2 factorial = 2! = 1*2 = 2

3 factorial = 3! = 1*2*3 = 6

4 factorial = 4! = 1*2*3*4 = 24

5 factorial = 5! = 1*2*3*4*5 = 120

6 factorial = 6! = 1*2*3*4*5*6 = 720

7 factorial = 7! = 1*2*3*4*5*6*7 = 5040

8 factorial = 8! = 1*2*3*4*5*6*7*8 = 40320

9 factorial = 9! = 1*2*3*4*5*6*7*8*9 = 362880

If you know the factorial of n-1, you can get n! by multiplying n-1! by n. Find 10!

n! = n*(n-1)!

(n*m)! ≠ n! x m!

(n + m)! ≠ n! + m!

If you have n different objects, they can be arranged in n! different arrangements.

How many different arrangements are possible of the letters in a) SAM; b) BLAKE.

a) 3 distinct letters. Answer= 3! = 6; b) 5 distinct letters. Answer = 5! = 120

How many different arrangements are possible of the letters in AIDAN?

There are two A's. You can swap the 2 A's in 2! =2 ways and get the same result.

So we divide 5! by 2! Answer = 5!/2! = 120/2 = 60.

How many different arrangements are possible of the letters in SAVANNAH?

There are 3 A's and 2 N's. The A's can be arranged in 3!=6 ways and N's in 2! ways.

Divide 8! by 3! and by 2!. Answer = 8!/(3!*2!) = 40320/(3*2) = 6720.

How many distinct arrangements can you make of the letters in MISSISSIPPI?

When we arrange things, the order matters in some situations and in other situations the order does not matter.

If you are making a salad with lettuce, onions, tomatoes and olives, the order in which you add the vegetables does not matter.

If you are spelling somebody's name, the order matters. SAM is different from MAS.

When order matters we have a **permutation**.

Let us say we want to elect a president, secretary and treasurer for the Math Club from a group of 8 students. The order in which we make the selection matters. John as President, Joe as Secretary, and Mary as Treasurer is different from Mary as President, Joe as Secretary and John as Treasurer, even though we picked the same three students.

We can select a President 8 ways, then a Secretary 7 ways, and a Treasurer 6 ways.

The number of permutations = 8x7x6 = 336

This is nothing but 8! divided by 5!. (8*7*6*5*4*3*2*1)/(1*2*3*4*5)

When we have a permutation of r things out of n, we represent it as **nPr**.

$$\mathbf{nPr} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!}$$

$$6\text{P}2 = \frac{6!}{4!} = 6 * 5 = 30$$

$$8\text{P}3 = \frac{8!}{54!} = 8 * 7 * 6 = 336$$

Your gym lock is a permutation lock, not a combination lock. A lock code of 479 is different from 947. Order matters!

With **combinations,** order does not matter. Suppose I want to select three students to bring cookies to the next meeting out of 8 Math Club students. Here the order does not matter. Picking Dave, then Josie and finally Anna is not different from picking Josie, then Anna and finally Dave. Since order does not matter and we can pick these three students in 3! = 6 ways, we must divide the number of permutations by r! (3!) to get the number of combinations represented as nCr.

$$\mathbf{nCr} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})! * \boldsymbol{r}!}$$

$$8C3 = \frac{8!}{5! * 3!} = 56$$

There are 9 desserts in front of you. You can pick 2. Is this a combination or permutation? How many different selections are there?

There are 8 new video games at GameStart. You wish to select a most favorite, second favorite and third favorite game? Is this a permutation or combination? How many different selections are possible?

**Refresher:**

**Prime numbers**

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, …

**Squares:**
1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, …

**Cubes:**

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, …

**Solutions:**

MISSISSIPPI: $\frac{\mathbf{11}!}{\mathbf{4}! * \mathbf{4}! * \mathbf{2}!}$ = 34650

Desserts: Combination: 9C2 = 36 dessert choices

Video games: Permutation: 8P3 = 336 selections